A variety of lepton number violating processes related to Majorana neutrino masses

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ABSTRACT

A Majorana type of the neutrino mass matrix induces a class of lepton number violating processes. Cross sections of these reactions are given in terms of the neutrino mass matrix element, and a semi-realistic event rate is estimated. These processes provide mass and mixing parameters not directly accessible by the neutrino oscillation experiments. If these processes are discovered with a larger rate than given here, it would imply a new physics of the lepton number violation not directly related to the Majorana neutrino mass, such as R-parity violating operators in SUSY models.

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Neutrino oscillation observed in SuperKamiokande [1], SNO [2], and Kam-LAND [3] experiments have opened a new window beyond physics of the standard model. The immediate critical question is to determine the nature of neutrino masses; whether they are of Dirac or of Majorana type. Unfortunately, the data on the neutrino oscillations can not discriminate between the types of neutrino masses, as we will see below. In order to get a conclusive argument on this issue, the investigation into lepton number violating processes, inevitable consequence of the Majorana nature, is indispensable.

If the Majorana mass is verified, it opens up the possibility of generating the baryon asymmetry of the universe via leptogenesis scenario [4]. A conventional lepton number violating process and the one most extensively discussed towards this goal is the neutrinoless double beta decay [5]. Despite several ingenious experimental proposals for improved detection of this decay we believe that some alternative methods to measure the Majorana type of the neutrino mass matrix are both useful and very important. In the present work we examine a variety of lepton number violating processes for this purpose.

We systematically examine a class of lepton number violating effective operators below the Fermi energy scale $1/\sqrt{G_F}$,

$$ll\,\bar{q}q\,\bar{q}q \, , \tag{1}$$

where l is the lepton doublet and q is the quark doublet having the quantum number of the standard model. Existence of this class of operators requires a new physics beyond the standard theory, but we do not need to specify the new physics. The Feynman diagram that generates this class of effective operators is depicted in Figure 1; the cross in the figure is the Majorana neutrino mass matrix and the exchanged particle is a weak boson, either $W^{\pm}$ or Z. When both of l are the electron and q is either a u- or d-type quark, the operator gives the neutrinoless double beta decay. We extend this
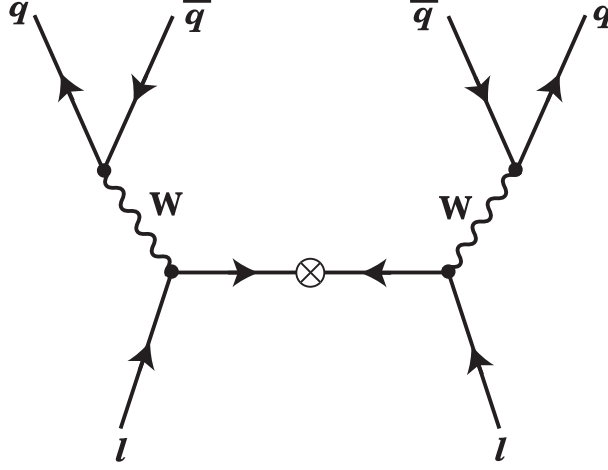
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Figure 1: The Feynman diagram that generates effective lepton number violating operators

double beta process to all combination of charged leptons of l, such that the full neutrino mass matrix element may be explored.

The strength of these dimension 9 operators at low energy scale of $\sqrt{s} \leq 250 GeV$ is of order

$$G_F^2 m_\nu s^{3/2} \sim 10^{-22} \frac{m_\nu}{1eV} \left(\frac{\sqrt{s}}{100MeV}\right)^3 . \tag{2}$$

This is an extremely small number compared to the usual weak interaction of order $G_F s$, but the nature of neutrino mass may well be examined only by these operators. The important point is that once the Majorana nature of the neutrino mass is assumed, there is no arbitrary freedom left, both for existence and its strength of the class of lepton number violating processes we consider below.

Cross sections of processes discussed below are determined by using the matrix element of the Majorana neutrino mass,

$$m_{\alpha\beta} = \Sigma_k U_{\alpha k} U_{\beta k} \, m_k , \tag{3}$$

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where $(k = 1, 2, 3)$ and $(\alpha, \beta = e, \mu, \tau)$ and m_k's are the mass eigenvalues (real and positive by definition). The reaction rate is thus independent of the origin of the neutrino mass matrix such as the seesaw mechanism [6]. Note that the above combination of neutrino masses and the mixing parameters is different from that measured in the neutrino oscillation experiment, in which $U_{\alpha k}$ and its conjugate $U_{\beta k}^*$ appears, thus the Majorana phases, characteristic of the Majorana nature, just cancel out in neutrino oscillation. Since the Majorana phase factor plays an important role in leptogenesis calculation, in general, the investigation into the lepton number violating reactions is relevant for the leptogenesis.

If one of these processes is discovered with a larger rate than given below, it means existence of a new class of diagrams not involving the neutrino mass matrix and may provide a new feature to the lepton sector such as R-parity violating interactions in SUSY models. In that sense lepton number violating processes complementary to the neutrinoless double beta decay may provide the unique window to determine the mechanism of how the lepton number violation occurs.

With the lepton flavor mixing, the effective operators above give a variety of lepton number violating processes. We consider the following reactions which we think relatively easy to explore experimentally;

$$(e\mu); \qquad e^- + A^Z \rightarrow \mu^+ + A^{Z-2} , \tag{4}$$

$$(\mu e); \qquad \mu^- + A^Z \rightarrow e^+ + A^{Z-2} , \tag{5}$$

$$(e\mu); \qquad p + A^Z \rightarrow \mu^+ + e^+ + (A+1)^{Z-1} , \tag{6}$$

$$(ee); \qquad e^- + e^-(atomic) \rightarrow \pi^- + \pi^- , \tag{7}$$

$$(e\mu); \qquad \nu_\mu + e^-(atomic) \rightarrow \pi^- + \pi^0 . \tag{8}$$

Here $(\alpha\beta)$ means that the process may explore the matrix element $m_{\alpha\beta}$. There are corresponding inclusive processes such as $e^- + A^Z \rightarrow \mu^+ + X$, where X is any hadronic state. When A is a light nucleus, it may break up

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due to a low binding energy, for instance,

$$e^- + He^4 \rightarrow \mu^+ + 4\,n \ . \tag{9}$$

To reduce the background at low energies to a respectable level, it is important to search for unambiguous signatures , for instance without decay muons below the π production threshold for the first process of (4). On the other hand, at high enough energies pions may not decay within the detector volume, in which case one does not worry about $\mu^\pm$ from the pion decay. It is also important to use a high intensity beam and a high density target in order to overcome the low reaction rate, for instance the atomic electron target in the processes of (7) and (8). We also consider muons captured by nucleus in (5) to compensate for the low flux. The other processes we considered and not listed here are difficult due to various reasons; the background problem, the insufficient beam energy available or planned now, a finite short lifetime and so on.

We consider important processes in turn, and the conversion processes, $e^- + He^4 \rightarrow \mu^+ + 4\,n$ and $e^- + A^Z \rightarrow \mu^+ + A^{Z-2}$ are the first. A related conversion process, $\mu^- + A^Z \rightarrow e^+ + A^{(Z-2)}$, $\mu^- + A^Z \rightarrow \mu^+ + A^{(Z-2)}$, has been considered in [5] and [7]. The experiment could be done effectively below the π-production threshold rejecting the μ^+ background from the π^+ decay, hence for this case the incident electron energy should be chosen properly. For the 2nd process of nuclear target, the cross section at low energies is given by

$$\sigma = \sum_f \sigma_{i \rightarrow f} = \frac{G_F^4 m_{e\mu}^2 <p_+><E_+>}{32\pi^3 R_n^2} \sum_{f,s} |t^{\mu\nu}\Omega_{\mu\nu}|^2 \ , \tag{10}$$

where $t^{\mu\nu}$ is the lepton wave function, $\Omega_{\mu\nu}$ is the nuclear matrix element and R_n is an effective nuclear radius, as defined in [8]. Since the final nucleus can take any final state, we sum up the final states. Therefore, we use the average momentum and energy for μ^+. We neglect the momentum of

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μ in $t^{\mu\nu}$ and use the approximation to evaluate the nuclear matrix element [8]. We find $\sum_{f,s} |t^{\mu\nu}\Omega_{\mu\nu}|^2 \simeq (g_V^2 + 18g_A^2 + 9g_A^4)Z(Z-1)$ for $S = 0$ and $(g_V^2 + 2g_A^2 + 9g_A^4)Z(Z-1)$ for $S = 1$. Here, S is the spin of two protons and Z is the atomic number. From this, we find with $Z = 50$ and $A = 100$

$$\sigma \sim 5 \times 10^{-65} \, cm^2 \, (\frac{|m_{e\mu}|}{100eV})^2 \ , \tag{11}$$

for the average μ momentum of $30MeV$. It may be worthwhile to comment that there is no reduction due to the nuclear matrix element in comparison with the double beta decay and also there is enhancement due to many possible combinations of the proton target inside the nucleus.

The helium case can roughly be estimated by just taking $Z = 2$, and we find

$$\sigma \sim 2 \times 10^{-67} \, cm^2 \, (\frac{|m_{e\mu}|}{100eV})^2 \ , \tag{12}$$

In order to compensate for the small reaction rate at low energies, it might be useful to go to higher energies despite larger backgrounds. Energy dependence of the cross section goes like $\propto s^2$ with $\sqrt{s}$ the CMS energy and it is $O[1] \, G_F^4 |m_{e\mu}|^2 \, s^2/16\pi^3$. A simple parton model neglecting transverse momentum may be used in estimating $O[1]$ factor here.

The advantage of muon capture listed in (5) is two fold; it automatically gives a self-focusing mechanism of the incident particle, capture into the area of nuclear size, and the same muon may repeatedly be used as in the case of a high luminosity accumulator ring. Thus, a naive computation yields a very large event rate, for instance even with a pulsed muon flux of order $10^{12}/sec$ much smaller than the highest achieved electron flux. The problem however is that at the same time the first order weak process, namely muon capture of $\mu^- \to \nu_\mu$ also becomes huge, and there is no chance of survival of captured muons left for the lepton number violating processes. Technically, the relevant quantity in this case is the branching fraction of muon captured reactions instead of the absolute event rate, and for this quantity

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the enhancement factor due to the capture into the atomic orbit cancels. For example, the branching fraction of the process $\mu^- \rightarrow e^+$ is given by $O[G_F^2|m_{\mu e}|^2 Z/8\pi^2 R^2] \sim 3 \times 10^{-29}|m_{\mu e}/100\text{eV}|^2$[5].

We next discuss the processes off atomic electrons, (7) and (8). Both processes are quite unique in that the lepton number disappears in the final state. For the atomic target the threshold for both processes opens up at around $\frac{2m_\pi^2}{m_e} \approx 80 GeV$. The cross section for the process $e^- + e^-$ is given by

$$\sigma = \frac{G_F^4 f_\pi^4 |m_{ee}|^2}{2\pi} \sqrt{\frac{s - 4m_-^2}{s}} \ , \tag{13}$$

where f_π is the π decay constant of order $90 MeV$, and $s \approx 2m_e E_e$ is the CMS energy squared. We have treated the atomic electron being at rest, since its momentum is much smaller than the incident beam energy $E_e (\gg m_e)$. The important prefactor $\frac{G_F^4 f_\pi^4 |m_{e\mu}|^2}{2\pi}$ is numerically $\approx 8 \times 10^{-67} \, cm^2 \, (|m_{e\mu}|/100eV)^2$. The event rate assuming an effective flux of order $10^{34} cm^{-2} sec^{-1}$ and a heavy target of mass $500g$ and $A \sim 2Z$ is $\approx 4 \times 10^2 \, (|m_{e\mu}|/100eV)^2/year$. A non-trivial background of the usual electromagnetic origin is $e^- + n \rightarrow e^- + p + \pi^- + \pi^- + \pi^+$ with a missing p. The detector should be arranged hermetically not to miss particles produced in the final state, and the kinematical condition of the two-body process of (7) should be maximally exploited to reject these backgrounds.

The corresponding inclusive process, $e^- + e^- \rightarrow X$, with X any hadronic state has a larger cross section of order, $10^{-4} G_F^4 |m_{ee}|^2 s^2$, or $\approx 10^{-58} \, cm^2 \, (|m_{ee}|/100eV)^2 \, (\sqrt{s}/100GeV)^4$ for $\sqrt{s} \leq 250GeV$. A possible experimental problem would be photons from π^0 decay which might be misidentified by electrons at high energies. Thus, it would be preferable, but difficult, to solely detect all charged modes not containing π^0 in the final state. Furthermore, for the atomic electron target a reasonable choice of the maximal $\sqrt{s}$ would be of order $1GeV$, and even for this one needs a TeV electron LINAC. A more precise formula of the inclusive cross section

may be computed by using the simple quark model of 3 colors, to give at $2m_W > \sqrt{s} \gg 2m_\pi$

$$\sigma = \frac{G_F^4 |m_{ee}|^2 s^2}{4\pi^5} \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \, f(x_1, x_2) \qquad (14)$$

where $\sqrt{s}x_i$ is the invariant mass of the quark pair, $\bar{u}_i d_i$. The integral of the invariant mass distribution $f(x_1, x_2)$ is about 0.07.

The neutrino beam is attractive from two reasons: the low background and a high intensity ν_μ beam may become available in the proposed neutrino factory. The high intensity ν_μ beam of energy of order $80 GeV$ may also be of interest due to the possibility of the τ-neutrino appearance experiment. A related tri-muon process to measure $m_{\mu\mu}$ has also been considered in [9], which belongs to another class of effective operators, $ll\bar{l}\bar{q}q$. Incidentally, we considered this class of operators along with yet another class of $ll\bar{l}l\bar{l}l$, searching all promising processes of the lepton number violation. Due to the experimental difficulty of determining the sign of the lepton number of neutrinos the process of [9], namely $\nu_\mu + p \rightarrow \mu^+ + \mu^+ + \mu^- + X$, seems the only feasible process of both of these two classes for signatures of unambiguous lepton number violation.

The cross section for (8) is just 1/4 times of the rate eq.(13), neglecting the pion mass difference. A non-trivial background comes from the usual neutral current weak process, $\nu_\mu + n \rightarrow \nu_\mu + p + \pi^- + \pi^0$ with a missing p. The true signal is kinematically selected by, for instance, plotting the invariant mass$((p_- + p_0)^2)$ distribution of two pions (low energy π^0 may also help by requiring two photon showers of $\pi^0 \rightarrow \gamma\gamma$ decay), which peaks around at $\sqrt{2m_e E_\nu}$ for a given incident neutrino energy E_ν.

Finally, a low energy proton beam may be used in (6). The cross section at low energies for this process is $O[10^{-70}]cm^2$ for $Z = 50$.

A meaningful limit of the absolute scale of neutrino masses is set by the current negative search of the neutrinoless double beta decay, which gives a

limit [10], for the combination,

$$|m_{ee}| = |\Sigma_k U_{ek}^2 m_k| < 0.3 eV \ . \tag{15}$$

This is a combination of the mass matrix element different from that appearing in the above process of (4), (6), (8), namely $m_{e\mu}$, but is identical to that of (7). For instance, with a phase cancellation the neutrinoless double beta decay and the related process (7) might be suppressed, but the other processes involving $m_{e\mu}$ might be relatively strong. On the other hand, WMAP has recently derived from a detailed fluctuation map of the microwave background the following limit [11], [12] $m_{max} < 0.2 eV$. If this limit is to be taken at a face value, it may not be necessary to set a neutrino mass limit of order $1 eV$ or larger. Nonetheless, we believe that terrestrial experiments are to be supplemented to exclude with certainty even this range of neutrino masses. Complementary and redundant information on neutrino mass matrix elements $m_{\alpha\beta}$ is crucial for further understanding of the lepton, hence the GUT sector. Thus, despite very small rates experimental search for these new reactions is very welcome.

A precise relation between the parameters of the neutrino oscillation and the lepton number violating process is given by,

$$\Sigma_\gamma m_{\alpha\gamma} m_{\beta\gamma}^* = \Sigma_k U_{\alpha k} U_{\beta k}^* m_k^2 . \tag{16}$$

The quantity in the right hand side can be determined by a precision data of the spectral distortion of K2K and Kamland oscillation experiments, when it is collaborated with the absolute mass scale determined by a direct mass measurement. The (absolute values of) $m_{\alpha\beta}$ in the left hand side, on the other hand, could be measured through the lepton number violating processes. In the physical observable in Eq.(16), however, the information of Majorana phases is lost, due to the product of mass matrix m and its hermitian conjugate $m^\dagger$. This is why we should study the lepton number violating processes,

which are caused by the mass matrix m itself containing the information of the CP violating Majorana phases.

To clarify this we write the unitary matrix U as

$$U = U_{MNS}P, \tag{17}$$

where U_{MNS} is the Maki-Nakagawa-Sakata matrix with 1 CP violating phase and $P = \mathrm{diag}(1, e^{i\alpha}, e^{i\beta})$, with α and β being CP violating Majorana phases. We easily find that the observable in (16) can be written as $(U_{MNS} \cdot \mathrm{diag}(m_1^2, m_2^2, m_3^2) \cdot U_{MNS}^\dagger)_{\alpha\beta}$, where the matrix P disappears. On the other hand, from Eq.(3) we realize that $m_{\alpha\beta}$ itself does depend on the Majorana phases:

$$m_{\alpha\beta} = (U_{MNS} \cdot \mathrm{diag}(m_1, m_2 e^{2i\alpha}, m_3 e^{2i\beta}) \cdot U_{MNS}^t)_{\alpha\beta}. \tag{18}$$

Having valuable information on U_{MNS} and the mass eigenvalues m_1, m_2, m_3 from the neutrino oscillation experiments, by use of Eq.(18) we can evaluate each of $|m_{\alpha\beta}|$ and therefore the feasibility of lepton number violating processes. In the approximation of neglecting θ_{13} and taking the maximal mixing for the atmospheric neutrino oscillation, the unitary matrix U_{MNS} is expressed by

$$U_{MNS} = \begin{pmatrix} c_\odot & s_\odot & o \\ -s_\odot/\sqrt{2} & c_\odot/\sqrt{2} & -1/\sqrt{2} \\ -s_\odot/\sqrt{2} & c_\odot/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}, \tag{19}$$

where $s_\odot = \sin\theta_\odot$ and $c_\odot = \cos\theta_\odot$. Substituting this expression for U_{MNS} in Eq.(18), we can readily compute $|m_{\alpha\beta}|$. We generally find that $|m_{\mu\mu}| = |m_{\tau\tau}|$ and $|m_{e\mu}| = |m_{e\tau}|$. To get further results, we consider three typical cases of neutrino mass eigenvalues, i.e., the hierarchical (H), the inverted hierarchical (IH) and the quasi-degenerate (QD) cases. For the case H, utilizing $m_3 \simeq \sqrt{\Delta m_{atm}^2} >> m_2 \simeq \sqrt{\Delta m_{sol}^2} >> m_1$, we find

$$|m_{ee}| \simeq s_\odot^2 \sqrt{\Delta m_{sol}^2},$$

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$$|m_{e\mu}| \simeq \frac{|s_{2\odot}|}{2\sqrt{2}}\sqrt{\Delta m^2_{sol}} \ ,$$

$$|m_{\mu\mu}| \simeq \frac{1}{2}\sqrt{\Delta m^2_{atm}} + \frac{1}{2}c^2_\odot c_{2(\alpha-\beta)}\sqrt{\Delta m^2_{sol}} \ ,$$

$$|m_{\mu\tau}| \simeq \frac{1}{2}\sqrt{\Delta m^2_{atm}} - \frac{1}{2}c^2_\odot c_{2(\alpha-\beta)}\sqrt{\Delta m^2_{sol}} \ . \tag{20}$$

Therefore, the m_{ee} which is the effective mass for the neutrinoless double beta decay is small and also the contribution of the Majorana phases is suppressed. For the IH case, by using $m_1 \simeq m_2 \sim \sqrt{\Delta m^2_{atm}} >> m_3$, we find

$$|m_{ee}| \simeq m_1\sqrt{1 - s^2_{2\odot}s^2_\alpha} \geq m_1|c_{2\odot}| \simeq \frac{1}{2}\sqrt{\Delta m^2_{atm}} \sim 0.03\text{eV} \ ,$$

$$|m_{e\mu}| \simeq \frac{|s_{2\odot}s_\alpha|}{\sqrt{2}}\sqrt{\Delta m^2_{atm}} \ ,$$

$$|m_{\mu\mu}| \simeq |m_{\mu\tau}| \simeq \frac{1}{2}|m_{ee}| \ . \tag{21}$$

In this case, the effective mass of neutrinoless double beta decay is bounded from below and the decay will be measurable in high precision experiments, thus providing a useful information on the Majorana phase α. For the QD case, by using $m_1 \simeq m_2 \simeq m_3 > \sqrt{\Delta m^2_{atm}}$, we find

$$|m_{ee}| \simeq m_1\sqrt{1 - s^2_{2\odot}s^2_\alpha} > 0.03\text{eV} \ ,$$

$$|m_{e\mu}| \simeq \frac{|s_{2\odot}s_\alpha|}{\sqrt{2}}m_1 \ ,$$

$$|m_{\mu\mu}| \simeq m_1\sqrt{1 - (s^2_\odot c^2_\odot s^2_\alpha + s^2_\odot s^2_\beta + c^2_\odot s^2_{\alpha-\beta})} \ ,$$

$$|m_{\mu\tau}| \simeq m_1\sqrt{1 - (s^2_\odot c^2_\odot s^2_\alpha + s^2_\odot c^2_\beta + c^2_\odot c^2_{\alpha-\beta})} \ . \tag{22}$$

Therefore, again we have an enough chance to measure the effective mass of the neutrinoless double beta decay. In addition, from the experiments to measure the other matrix elements we may be able to obtain the information on both of two Majorana phases α and β.

We thus learn that if the neutrinoless double beta decay unfortunately ends up with null result of precision of $O[0.03]$eV, only the normal hierarchical

mass pattern is allowed. In this case the lepton number violation can be verified in terrestrial laboratories only by lepton number violating processes discussed in the present work.

The interesting question is how CP violation, relevant for the leptogenesis, may be examined by studying the CP violating phases in the Majorama mass matrix of left-handed neutrinos, which are obtainable from the neutrino oscillation experiment, handled by U_{MNS}, and lepton number violating processes, as discussed above. To get the answer, we assume the seesaw mechanism as the origin of the Majorana neutrino masses. In this mechanism the relevant quantity for the leptogenesis is $m_D^\dagger m_D$, with m_D being the Dirac mass matrix in the basis where the Majorana mass matrix of right-handed neutrinos is diagonalized. The Dirac mass matrix m_D can be expressed as $m_D = U\sqrt{D_\nu}R\sqrt{D_R}$, where U is the mixing matrix given in Eq.(17), D_ν and D_R are the diagonal matrices whose eigenvalues are the observable small Majorana masses of left-handed neutrinos and the right-handed Majorana masses, respectively¿. R is a complex "orthogonal" matrix, $R^T R = 1$, with 3 independent CP phases, which is otherwise arbitrary. It may be worthwhile to observe $m_D^\dagger m_D = \sqrt{D_R}RD_\nu R^\dagger\sqrt{D_R}$. That is, the mixing matrix U is not directly related to the leptogenesis. We realize that not only the phase in U_{MNS}, but also the Majorana phases disappear in $m_D^\dagger m_D$. It, however, will be generally possible to relate the CP violating phases in U to those in R, responsible for the leptogenesis, once some complementary information, such as the absolute values of the elements of m_D, is obtained. It may also be possible that a relation between the phases of U and those of R is naturally realized in a sophisticated concrete model of neutrino mass generation.

A road map towards experimental verification of leptogenesis may not be easy to draw. A complete determination of the Majorana mass matrix including the CP phase may require a variety of lepton number violating reactions such as $nn \rightarrow eepp$ (neutrinoless double beta process), $e^- \rightarrow \mu^+$,

$\nu_\mu \to \mu^- \mu^+ \mu^+$, and $pp \to \tau^+ \tau^+ nn$. Even this much is not sufficient; one further needs a handle to the mass scale of the heavy Majorana particle in the seesaw mechanism to compare with leptogenesis calculation.

We finally make a short comment on processes not discussed so far. The binding system of stable atoms (e^- + nucleus) has been considered since in this case a shorter time scale different from the reaction time is available (like in the case of neutrinoless double beta decay), but we did not find a useful system due to the difficulty of overriding the barrier of nuclear binding. For instance, the He atom cannot spontaneously decay emitting e^+. As to the decay process, $\tau^- \to \mu^+ + X$, $B^+ \to \mu^+ + \mu^+ + X$ etc. with X any hadronic state is conceivable, which has however a branching ratio of order, $10^{-4} (G_F |m_{\mu\tau}| m_\tau)^2 \approx 10^{-18} (|m_{\mu\tau}|/100 eV)^2$ for τ, too small even for the tau-charm factory. (The current upper limit is of order 10^{-6}.)

Thus, to the best of our knowledge, the processes discussed in the present work are the best candidates to explore the Majorana nature and its strength of the neutrino mass matrix, beyond the neutrinoless double beta decay. If these tiny rates are experimentally falsified by larger rates, it definitely implies a new source of lepton number violation besides the Majorana neutrino mass.

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